USE OF RELAXATION VISCOELASTIC MODEL IN CALCULATING UNIAXIAL HOMOGENEOUS STRAINS AND REFINING THE INTERPOLATION EQUATIONS FOR MAXWELLIAN VISCOSITY

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The work applies the relaxation viscoelastic model proposed in [1-3] for calculations of high-rate deformation of bars and plates and for refining the interpolation equations of Maxwellian viscosity χ (a magnitude inverse to the relaxation time τ of the tangential stresses) by means of them. These calculations were carried out to study the dependence of the dynamic yield limit σ_g on the deformation rate $\dot{\varepsilon}$. The authors propose that the dependence $\sigma_g(\dot{\varepsilon}, T)$ in $\dot{\varepsilon}$ be transformed, such that $\chi = \dot{\varepsilon}(\sigma, T)$ (here σ is the intensity of the tangential stresses and T is temperature) in order to construct interpolation equations for Maxwellian viscosity χ. A numerical analysis demonstrated that this equation leads to the correct qualitative dependence in calculations of $\sigma_g(\dot{\varepsilon})$. A correction factor is introduced into the equation $\chi = \chi(\sigma, T)$ in order for the numerical calculations to quantitatively coincide with the experimental data in this work.

Let us consider the uniaxial deformation of a bar of length L in the direction of the ox axis. The left end of the bar is fastened at the point $x_0 = 0$, while the right end is deformed at a rate U (t), i.e., $x_1 = L + \int_0^t U(t)\,dt$. The velocity of points of the bar is linearly distributed along the length of the bar in homogeneous deformation, i.e.,

$$u(x,t) = U(t)x/x_1(t),$$

from which we find that the deformation rate $\dot{\varepsilon}$ has the form

$$\dot{\varepsilon} = \frac{\partial u}{\partial x} = \frac{U(t)}{x_1(t)} = \frac{U(t)}{L + \int_0^t U(t)\,dt}. \tag{1}$$

In all cases that have been studied,

$$\frac{\Delta L}{L} = \frac{\int_0^t U(t)\,dt}{L} \ll 1, \text{ i.e., } \dot{\varepsilon} \approx U(t)/L.$$

Suppose the oy and oz axes are situated perpendicular to the direction of deformation of the bar. The equation of state of the bar material has the form [2]

$$E = E(\alpha, \beta, \gamma, S).$$

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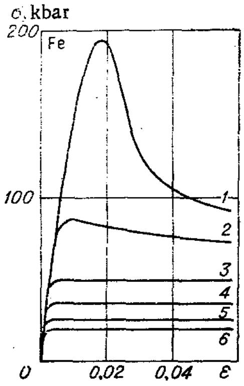


Fig. 1

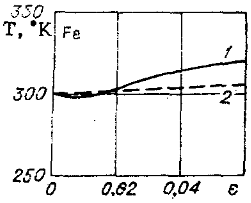
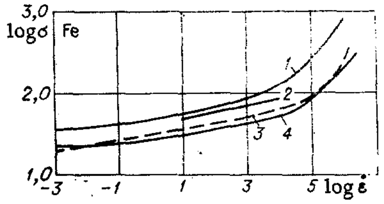


Fig. 2



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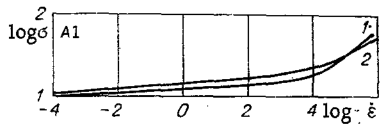

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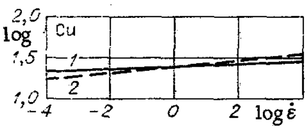
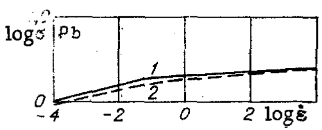


Fig. 5



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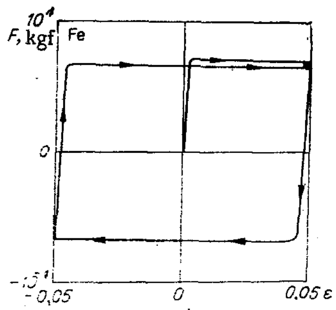

Fig. 7

Here α, β and γ are the logarithms of the relative elongations of the bar along the ox, oy, and oz axes, respectively, and S and E are the entropy density and energy per unit of mass. The stresses σ_x, σ_y, and σ_z along the ox, oy, and oz axes are determined by the equation [1]

$$\sigma_x = \rho \frac{\partial E}{\partial \alpha}; \quad \sigma_y = \rho \frac{\partial E}{\partial \beta}; \quad \sigma_z = \rho \frac{\partial E}{\partial \gamma}, \tag{3}$$

while the temperature is determined by the equation

$$T = \frac{\partial E}{\partial S}, \tag{4}$$

where ρ is the density of the substance. We will assume, by letting the bar be thin, that

$$\sigma_y \equiv \sigma_z \equiv 0. \tag{5}$$

over the entire length of the bar. Since the problem is symmetric in the yz plane, while the substance is isotropic,

$$\beta \equiv \gamma. \tag{6}$$

All the magnitudes are functions solely of time in homogeneous deformation. Under this assumption, the equation [1] describing the deformation of the bar takes the form

$$\begin{cases} \frac{d\alpha}{dt} = \dot{\varepsilon} - \left(\alpha - \frac{\alpha+\beta+\gamma}{3}\right)\chi; \\ \frac{dS}{dt} = \frac{4b^2}{T} D\chi \end{cases} \tag{7}$$

[we must add Eqs. (1)-(6) to these equations]. Here b is the velocity of propagation of transverse waves; $D = \frac{1}{2}\left[\left(\alpha - \frac{\alpha+\beta+\gamma}{3}\right)^2 + \left(\beta - \frac{\alpha+\beta+\gamma}{3}\right)^2 + \left(\gamma - \frac{\alpha+\beta+\gamma}{3}\right)^2\right]$ is the quadratic invariant of the Hencky tensor deviator, and $\chi = \chi(\sigma, T)$ is the magnitude of Maxwellian viscosity [3], where $\sigma = \sqrt{\tfrac{1}{2}[(\sigma_x - \sigma_y)^2 + (\sigma_y - \sigma_x)^2 + (\sigma_z - \sigma_x)^2]}$ is the intensity of the tangential stresses. The magnitude b is calculated from Eq. (2) using the equation

$$b = \sqrt{\left(\frac{\partial E}{\partial D}\right)_{\rho S}}.$$

Whenever deformation of a thin plate situated in the xy plane and dilated (contracted) along the x axis is considered, Eqs. (5)-(7) must be replaced by the equation

$$\begin{cases} \frac{d\alpha}{dt} = \dot{\varepsilon} - \left(\alpha - \frac{\alpha+\beta+\gamma}{3}\right)\chi, \\ \frac{d\gamma}{dt} = -\left(\gamma - \frac{\alpha+\beta+\gamma}{3}\right)\chi, \\ \frac{dS}{dt} = \frac{4b^2}{T} D\chi \end{cases}$$

and the relation

$$\sigma_z = 0.$$

Figure 1 depicts curves for the dependence of the magnitude $\sigma = -\sigma_x$ for bar deformation on relative elongation $\varepsilon = \Delta L/L$ for different deformation rates $\dot{\varepsilon}$. [The material was soft iron (α-phase) and σ is in kg/mm^2]. Curves 1-6 correspond to $\dot{\varepsilon} = 10^6$, 10^5, 10^3, 10, 10^{-1}, and 10^{-3} sec^{-1}. The initial temperature T_0 of the specimens was 300°K. The drop on curves 1 and 2 is due to heating of the specimen at high plastic deformations. Figure 2 depicts temperature curves $T(\varepsilon)$ at $\dot{\varepsilon} = 10^6$ sec^{-1} and $\dot{\varepsilon} = 10$ sec^{-1} (curves 2 and 1). We may conclude from the form of the curves in Fig. 1 that the stress in the specimen is less some maximum values σ_{cr} in the deformation process; it may be reasonably correlated with the yield limit σ_g, which can be experimentally observed. The dependence $\log \sigma_{cr} (\log \dot{\varepsilon})$ obtained by means of a numerical calculation (curve 1) and curve 4 for the dependence $\log \sigma_g (\log \dot{\varepsilon})$ taken from a previous experiment [3] are given in Fig. 3. Iron was the material of the specimens, the initial temperature of the specimens was 300°K, the magnitudes of the stresses were taken in kg/mm^2, and that of $\dot{\varepsilon}$ in sec^{-1}. The nature of these curves allows their qualitative coincidence to be judged.

We will introduce the correction factor $^3\rho_0 b_0^2/\sigma$ (here ρ_0 and b_0 are the density and velocity of propagation of the transverse waves under normal conditions) in the formula for Maxwellian viscosity $\chi(\sigma, T)$ [3] in order the dependences $\log \sigma_g = f(\log \dot{\varepsilon})$ and $\log \sigma_{cr} = f(\log \dot{\varepsilon})$ to quantitatively coincide. Then $\chi(\sigma, T)$ [3] for metals (iron, aluminum, copper, and lead) will be given by the equations

$$\chi = \chi_0 \left(\frac{\sigma}{\rho_0 b_0^2} q\right)^{n(T)-1} \exp(-\mu U(\sigma, T)/RT); \tag{8}$$

$$n(T) = \left[n_0 \left(\frac{T}{\theta_0} - n_1\right)^2 + n_2\right]^{-1};$$

$$U(\sigma, t) = c_0^2 (n(T) F(T) \pm \Phi(\sigma)).$$

The minus sign is taken for lead and the plus for the other metals:

$$F(T) = (F_0 - F_1 T/\theta_0) T/\theta_0;$$

$$\Phi(\sigma) = \Phi_0[\varphi(\sigma) - \sqrt{\varphi^2(\sigma) + \Phi_1}];$$

$$\varphi(\sigma) = \varphi_0 \ln(\sigma q/\rho_0 c_0^2) + \varphi_1.$$

TABLE 1

	χ_0, sec^{-1} 10^5	ρ_0, g/cm^3	c_0, km/	θ', K	μ, g/mole
Fe	0,0683	7,84	5,694	420	55,85
Al	0,0243	2,785	6,125	390	26,98
Cu	0,0417	8,90	4,651	315	63,54
Pb	0,0740	11,34	2,151	88	207,21

TABLE 2

	Fe	Al	Cu	Pb
q	$2{,}6\cdot10^4$	$1{,}06\cdot10^4$	$1{,}96\cdot10^4$	$0{,}535\cdot10^4$
n_0	0,0434	0,0462	0,0202	0,00804
n_1	1,545	2,57	0,955	0
n_2	0,03	0,01	0,035	0,01
F_0	$7{,}12\cdot10^{-3}$	$1{,}18\cdot10^{-2}$	$7{,}15\cdot10^{-3}$	$2{,}06\cdot10^{-3}$
F_1	$1{,}89\cdot10^{-3}$	$4{,}77\cdot10^{-3}$	$0{,}99\cdot10^{-3}$	$0{,}377\cdot10^{-3}$
Φ_0	$1{,}37\cdot10^{-3}$	$3{,}19\cdot10^{-3}$	0	$2{,}6\cdot10^{-3}$
Φ_1	14,15	53,1	0	10,15
φ_0	7,85	21,25	0	14,9
φ_1	—32,5	—59,7	0	—9,1

Here μ is molecular weight, θ_0 is the Debye temperature, c_0 is the velocity of longitudinal waves under normal conditions, and $R = 8\cdot31\cdot10^7$ ergs/deg·mole. The values of the magnitudes ρ_0, c_0, θ_0, χ_0, and μ are presented in Table 1 and the interpolation constants q, n_0, n_1, n_2, F_0, F_1, Φ_0, Φ_1, φ_0, and φ_1 are given in Table 2.

The calculation for $\log\sigma_{cr} = f(\log\dot{\varepsilon})$ using Eq. (8) for $\chi(\sigma, T)$ demonstrates that the dependence $\log\sigma_{cr} = f(\log\dot{\varepsilon})$ and $\log\sigma_g = f(\log\dot{\varepsilon})$ quantitatively coincide. This curve 3 recalculated from curve 1 using the refined Eq. (8) is presented in Fig. 3 for iron. Only the curves in Figs. 1 and 2 were obtained using these equations.

Results from calculations for Al, Cu, and Pb are presented in Figs. 4-6. Curves 1 and 2 of the figures correspond to curves 3 and 4 of Fig. 3, i.e., the dependences $\log\sigma_{cr} = f(\log\dot{\varepsilon})$ and $\log\sigma_g = f(\log\dot{\varepsilon})$, respectively ($\sigma$ is in kg/mm^2 and $\dot{\varepsilon}$ is in sec^{-1}).

Deformation curves calculated for plates are of the same qualitative form as for bars, differing from them only insignificantly. As a result of numerical calculations we were able to establish that the magnitudes σ_{cr} computed for the bar and calculated for a plate differ by about 10% in the range of deformation rates up to $\dot{\varepsilon} = 10^4$ sec^{-1}. Figure 3 depicts the dependence $\log\sigma_{cr} = f(\log\dot{\varepsilon})$ calculated for the deformation of an iron plate at $T_0 = 300$°K (curve 2).

It is of interest to calculate one deformation cycle of a bar with periodic rate U (t). Figure 7 depicts the curve for the dependence of longitudinal force F acting on an iron bar with initial cross section 1 cm^2 on the relative elongation ε. Deformation occurred at a rate $\dot{\varepsilon} = 10$ sec^{-1}. The form of the curve indicates the presence of a "Bauschinger-type" effect and σ_{cr} differs for different cycles.

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